Post-quantum Security of the Sponge Construction

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Post-Quantum Cryptography

- Search for cryptography secure against quantum adversaries
  - Public-key (based on new problems)
  - Symmetric-key (double security parameter)

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- **Are models, constructions and proofs still sound in the post-quantum setting?**

Motivation

- Imagine a commitment scheme using hash function H

Canonical Commitment Scheme

1. $r \xleftarrow{\$} R$
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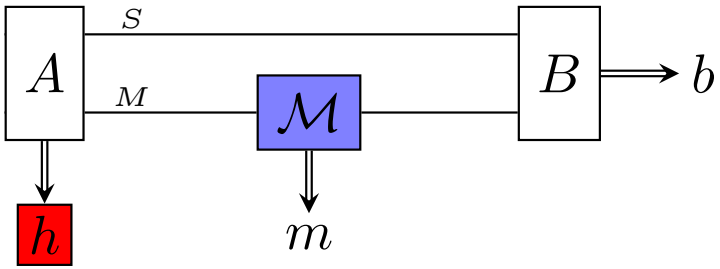
- Binding: cannot come up with two different openings to the same commitment
- Post-quantum we need something stronger[1]: **collapse binding**

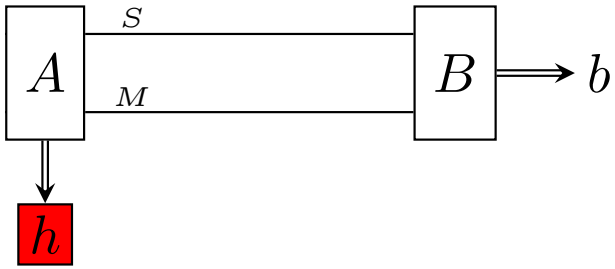
[1] D. Unruh - Computationally binding quantum commitments [Eurocrypt 2016]

Collapsing functions

- Adversary (A, B) has quantum oracle access to H



(a) – Game_1



(b) – Game_2

- In Game_2, B receives $M = |\phi\rangle = \sum_{\substack{m \\ H(m)=h}} \alpha_m |m\rangle$
- In Game_1, B receives $M = |m\rangle$ with probability $|\alpha_m|^2$
- Purely a quantum definition!

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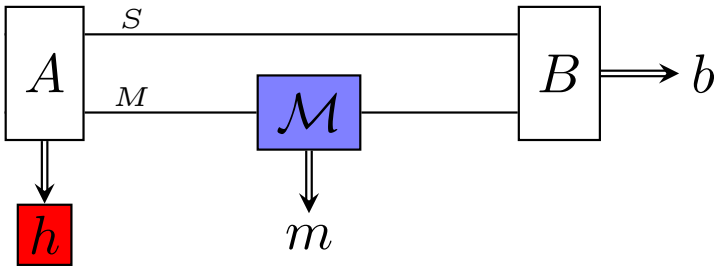

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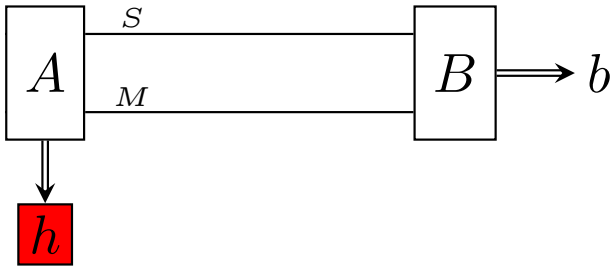

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- H collapsing iff $\mathbb{P}[b = 1 : \mathrm{Game}_1] \approx \mathbb{P}[b = 1 : \mathrm{Game}_2]$
- Intuitively: register M can be measured without (A, B) noticing

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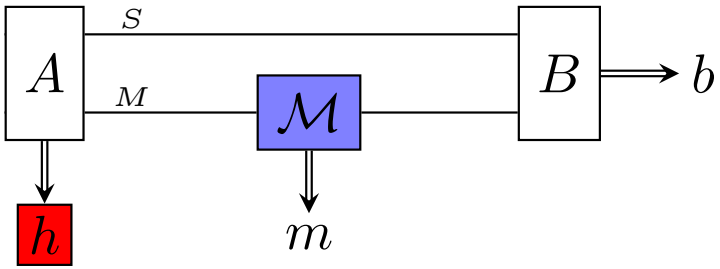

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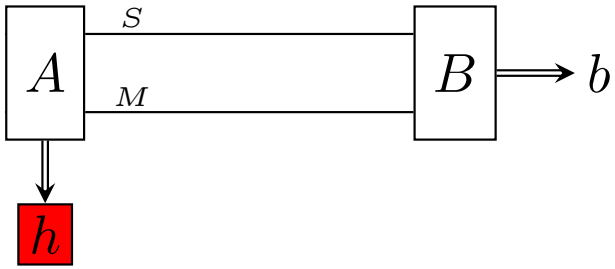

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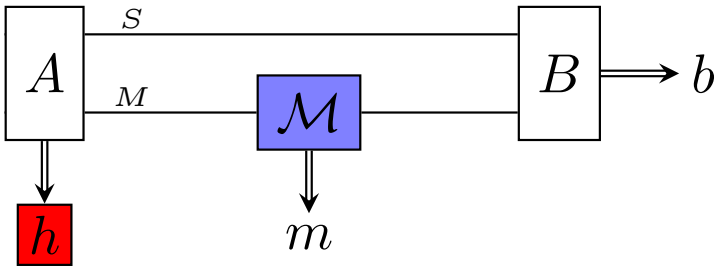

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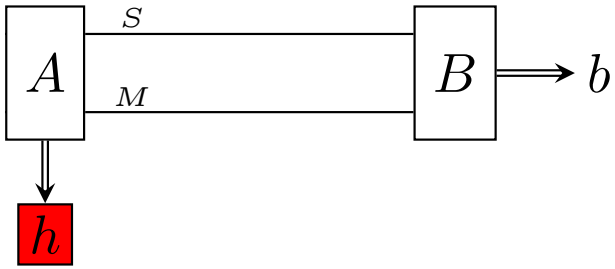

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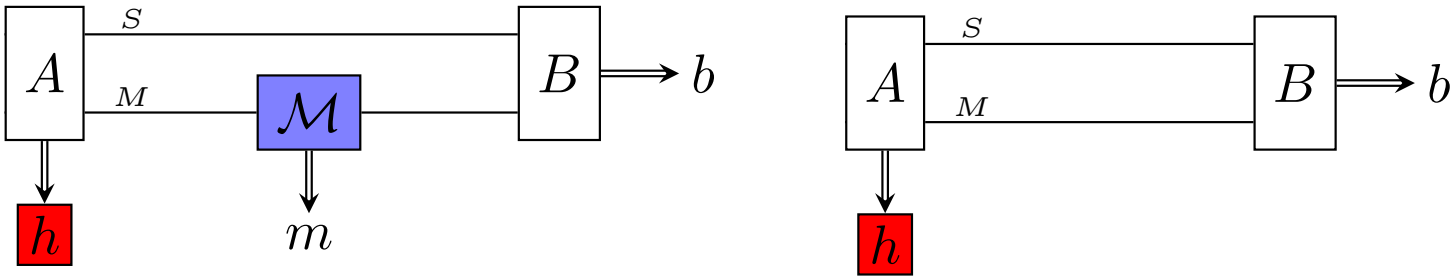

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- More interesting example: random oracles

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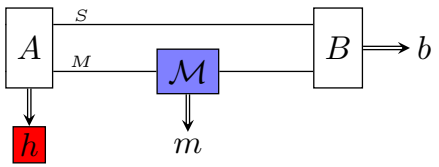

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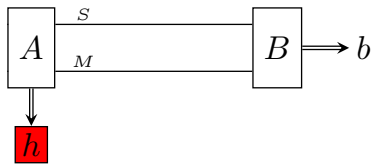


(b) – Game_2

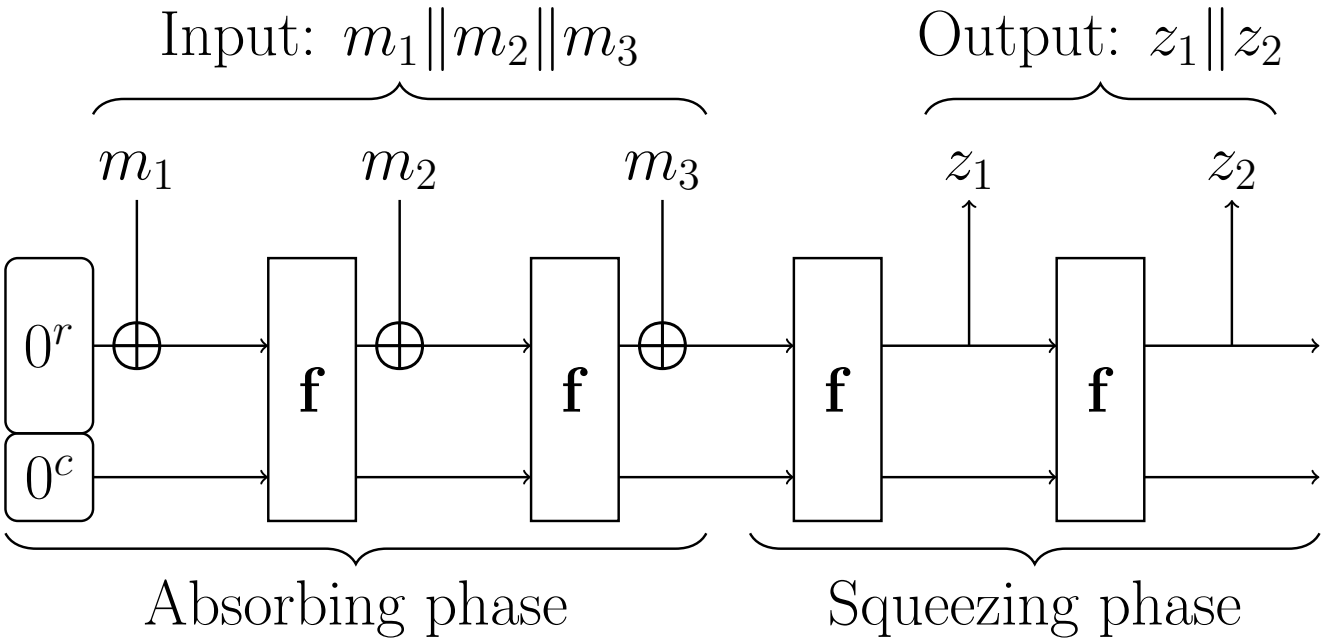
- In Game_1, M collapses to one of the states so $\mathbb{P}[b = 1 : \text{Game}_1] = \frac{1}{2}$
- In Game_2, M stays $|\phi\rangle$ so $\mathbb{P}[b = 1 : \text{Game}_2] = 1$

Hash functions

- Hash-functions: virtually used everywhere
- Post-quantum we want hash-functions to be collapsing
- Unruh[2] showed Merkle-Damgard is collapsing (SHA2)
- What about other constructions for hash-functions?

[2] D. Unruh - Collapse-binding quantum commitments without random oracles [Asiacrypt 2016]

Sponge construction



- For example Keccak(SHA3)

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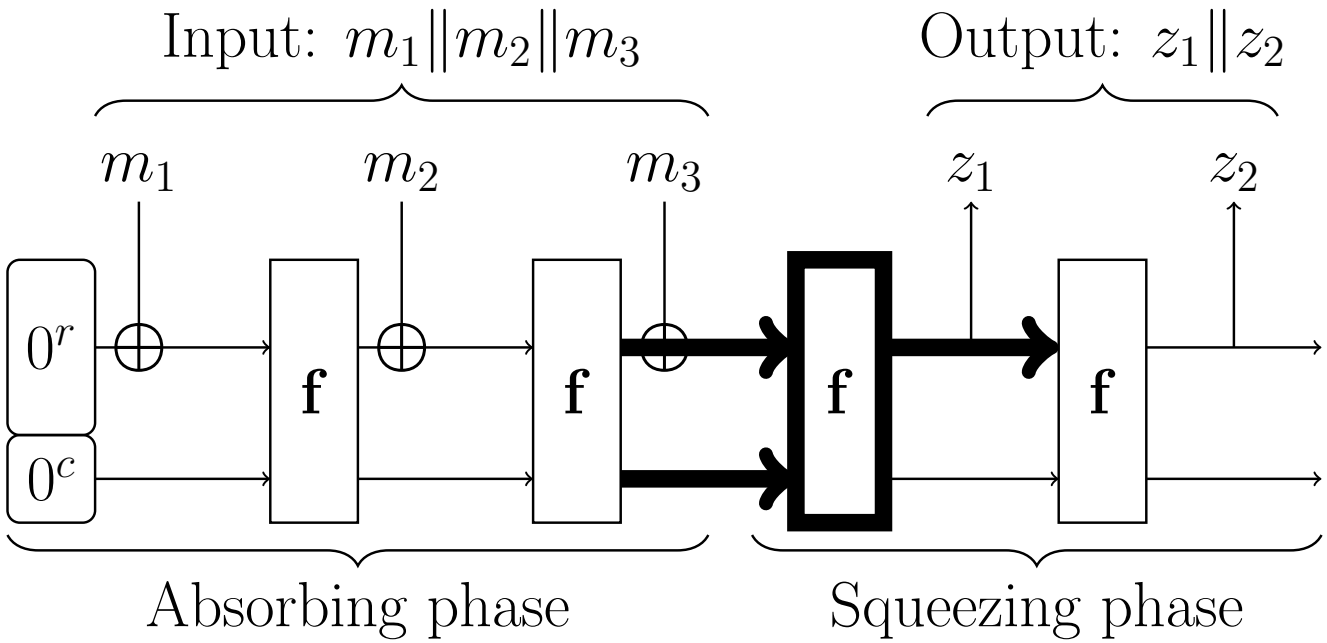

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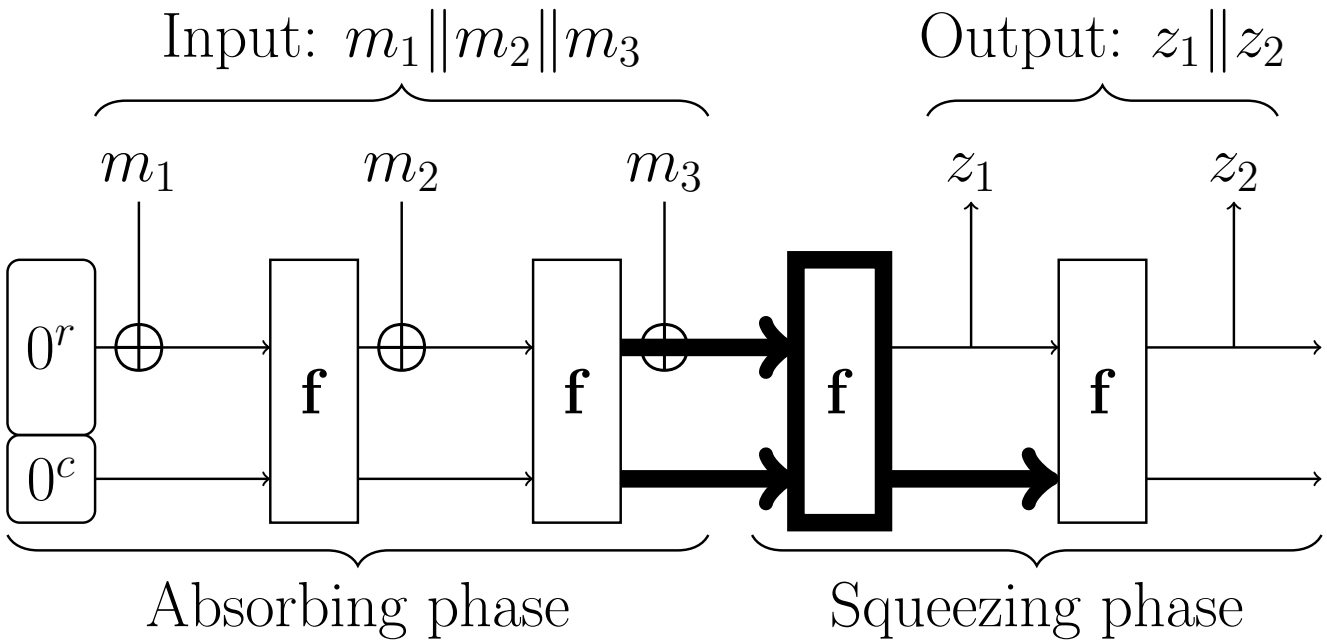

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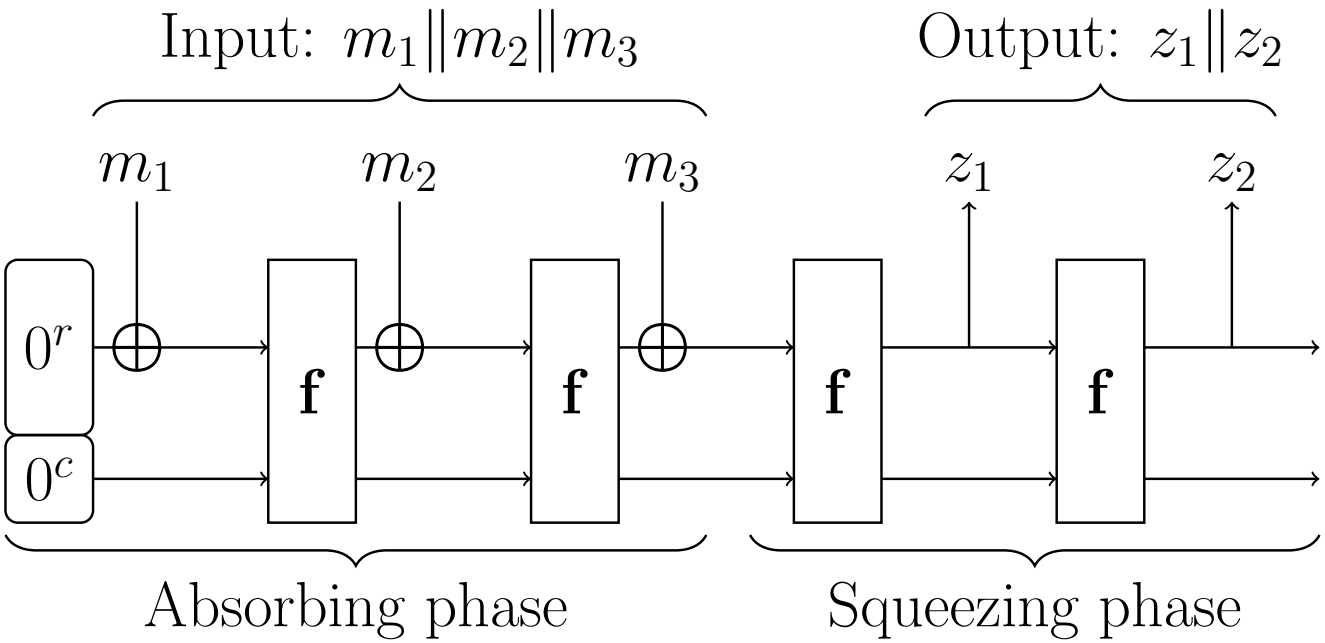

- Internal block-function $\mathbf{f} := \mathbf{f}^{\text{left}} || \mathbf{f}^{\text{right}}$ with $\mathbf{f}^{\text{left}} : \{0,1\}^{r+c} \to \{0,1\}^r$, $\mathbf{f}^{\text{right}} : \{0,1\}^{r+c} \to \{0,1\}^c$

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Requirements on internal block-function

- We only want to make assumptions on $\mathbf{f}$
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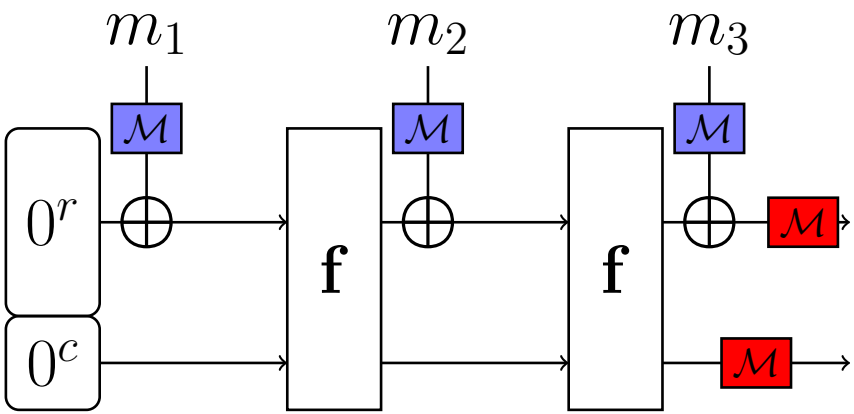
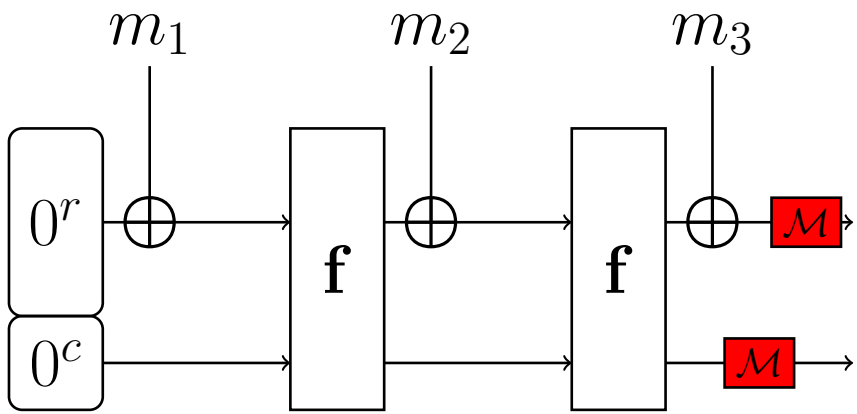
- pad is injective, thus it is collapsing
- If g, h are collapsing, so is $g \circ h$
- To show: $\mathbf{S}^{out}$ and $\mathbf{S}^{in}$ collapsing
- Next: proof sketch for collapsing of $\mathbf{S}^{in}$

Proof sketch $\mathbf{S}^{in}$ collapsing



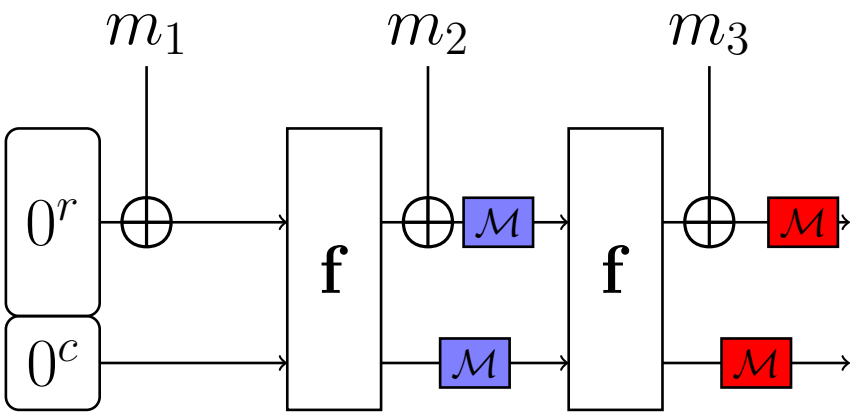
(a) $\mathrm{Game}_1 = \mathrm{Hyb}_1$



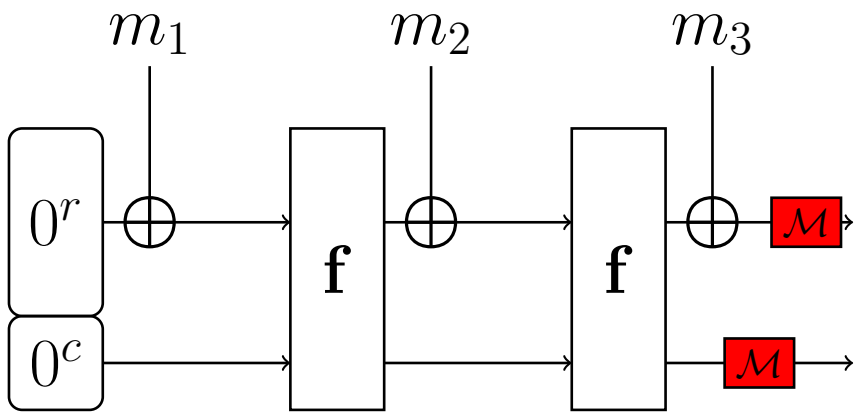
(b) $\mathrm{Game}_2 = \mathrm{Hyb}_4$

- Hybrid argument: subsequently measure more using properties of $\mathbf{f}$

Proof sketch $\mathbf{S}^{in}$ collapsing

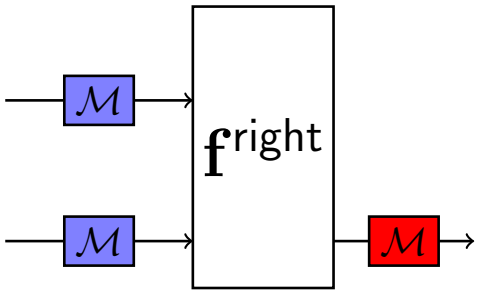


(a) Hyb_3

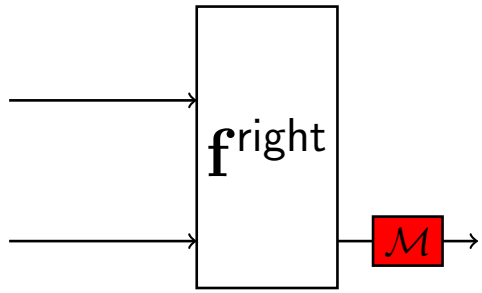


(b) Hyb_4

- Hybrid argument: subsequently measure more using properties of $\mathbf{f}$
- Collapsing games for $\mathbf{f}^{\mathrm{right}}$

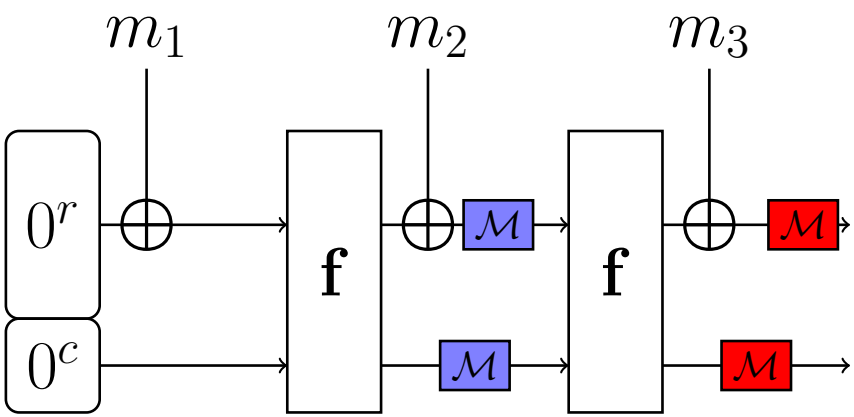


(c) Game_1



(d) Game_2

Proof sketch $\mathbf{S}^{in}$ collapsing



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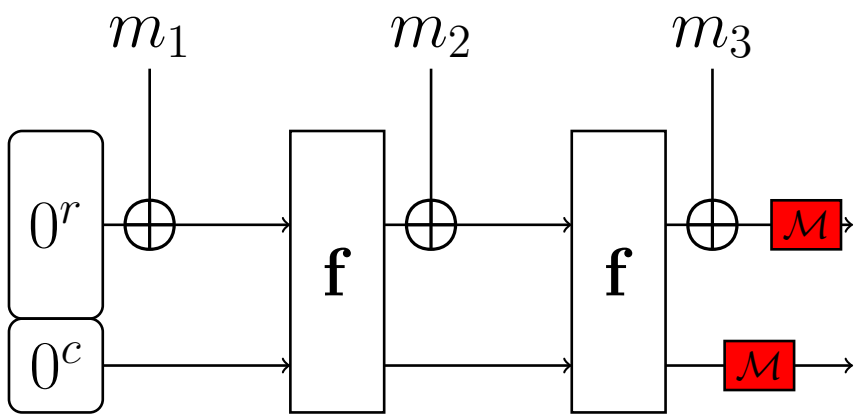
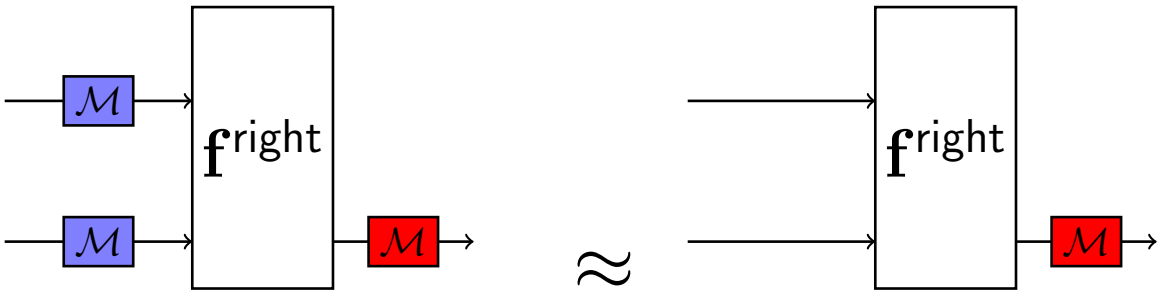


(b) Hyb_4

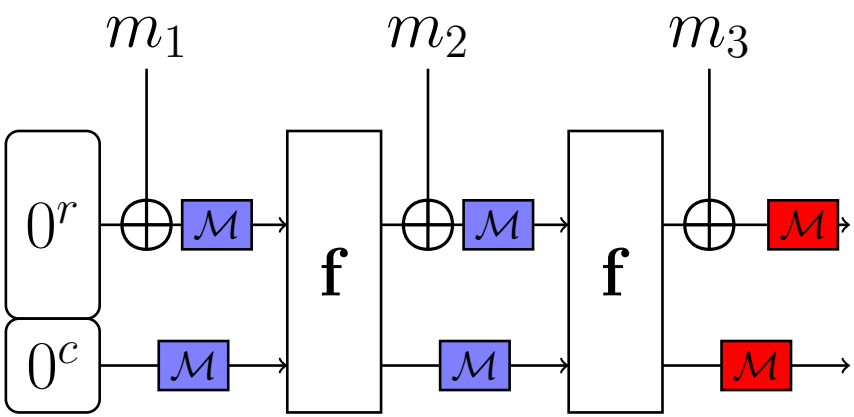
- Required property: $\mathbf{f}^{\mathrm{right}}$ collapsing



(c) Game_1 (d) Game_2

- $\mathbb{P}[b = 1 : \mathrm{Hyb}_3] \approx \mathbb{P}[b = 1 : \mathrm{Hyb}_4]$

Proof sketch $\mathbf{S}^{in}$ collapsing



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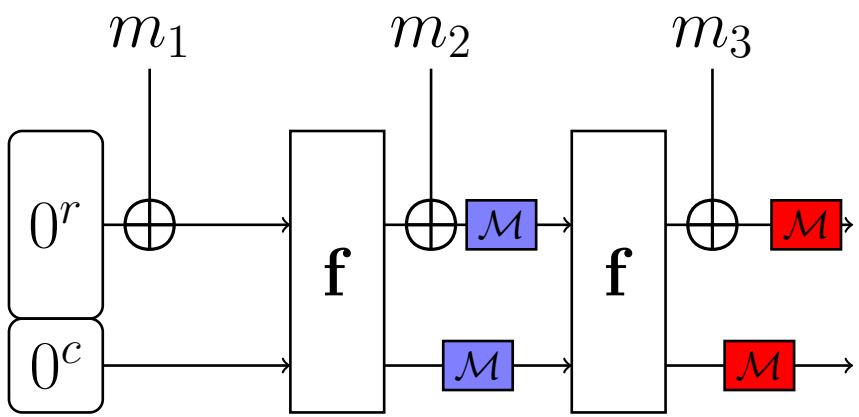

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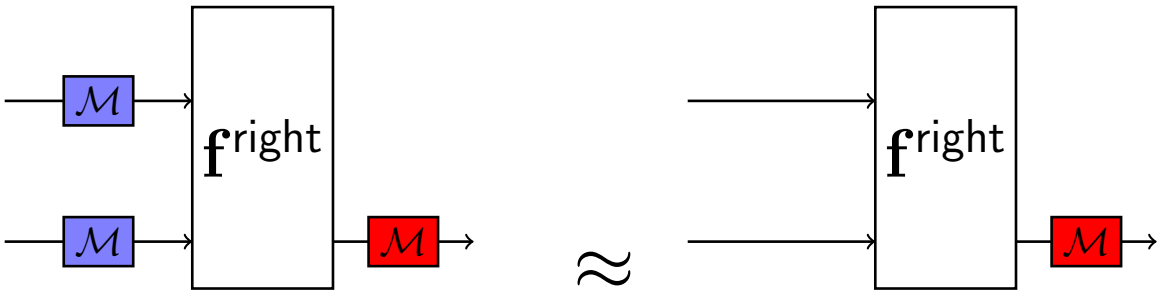

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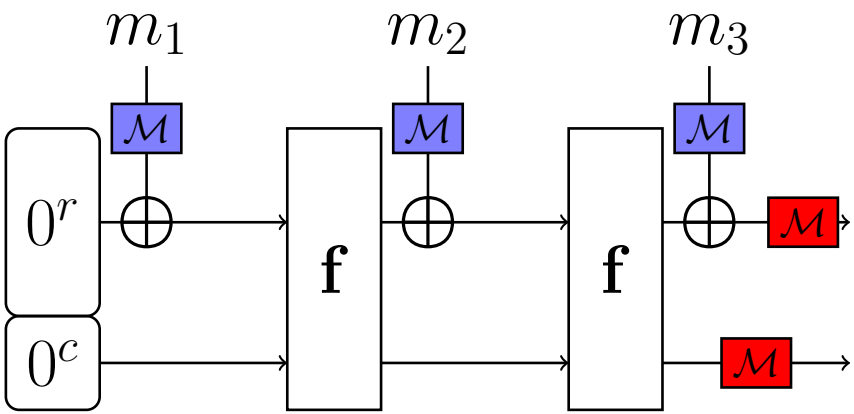

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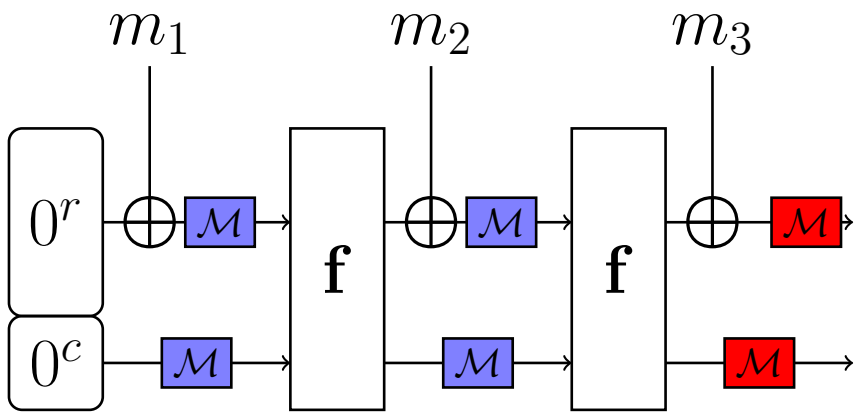

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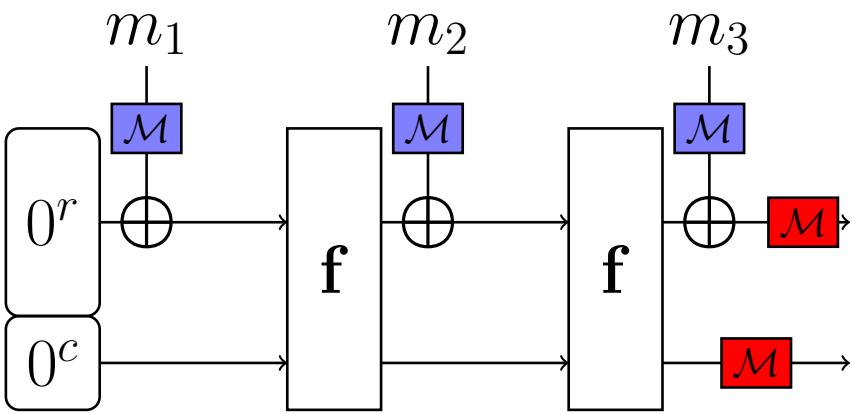

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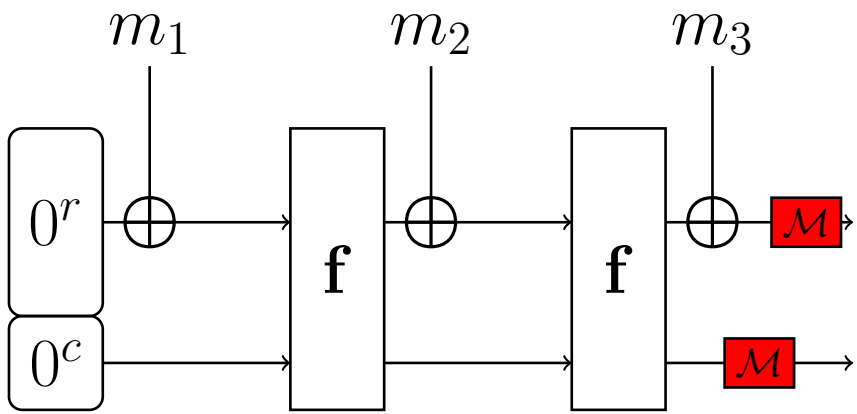

(b) $\mathrm{Game}_2 = \mathrm{Hyb}_4$

- Careful: this gives the misleading impression that all messages in superposition are of equal length
- Additional requirement: $\mathbf{f}^{\mathrm{right}}$ zero-preimage resistant

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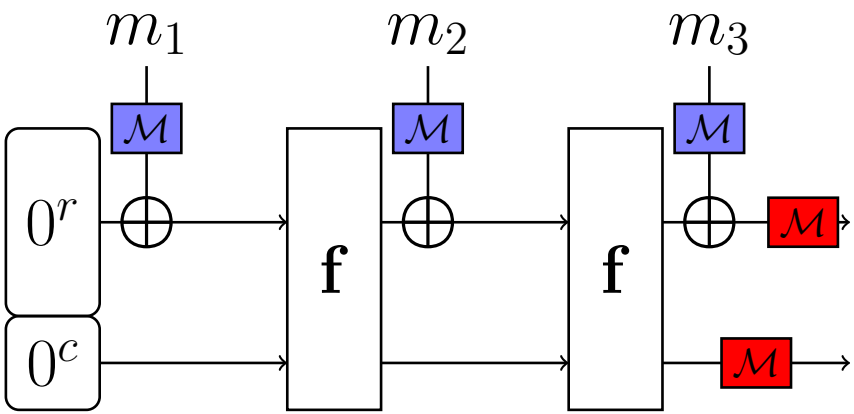

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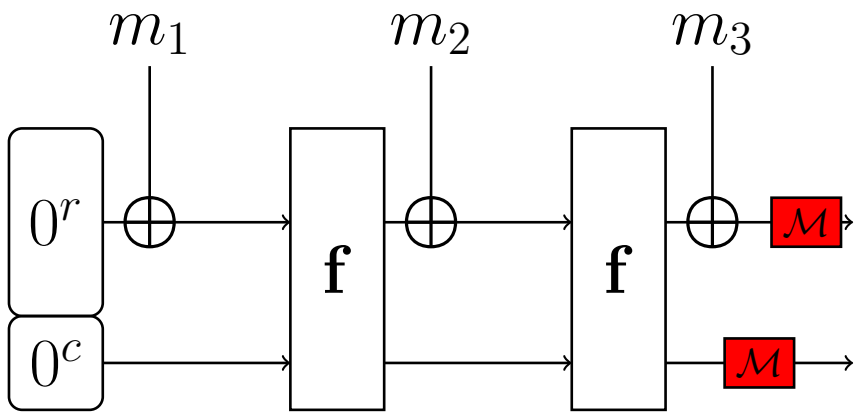

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- Concluding: $\mathbb{P}[b = 1 : \mathrm{Game}_1] \approx \mathbb{P}[b = 1 : \mathrm{Game}_2]$ if $\mathbf{f}^{\mathrm{right}}$ collapsing and zero-preimage-resistant

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- However, Keccak is an efficiently invertible permutation! $\rightarrow$ $\mathbf{f}^{\text{left/right}}$ not collapsing nor zero-preimage-resistant

A lot more in the paper!

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Questions?