# Flush, Gauss, and Reload

## A Cache-Attack on the BLISS Lattice-Based Signature Scheme

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# Lattice-based Cryptography

- Lattice-based cryptography: promising post-quantum secure alternative
- Active research on theoretical and practical security
- But what about security of implementations?

# This work

- The first side-channel attack on a lattice-based signature scheme
- Exploits information leakage from the discrete Gaussian sampler via cache memory
- Attack target: BLISS, an efficient lattice-based signature scheme

# BLISS

# BLISS Lattice-based Signature Scheme

- Bimodal Lattice Signature Scheme (BLISS) (CRYPTO '13 by Léo Ducas and Alain Durmus and Tancrède Lepoint and Vadim Lyubashevsky)
- Implementations available via NTRU lattices (polynomials in $R_q = \mathbb{Z}_q[x]/(x^n + 1)$, $n = 2^r$, prime $q$).
- For $f, g \in R_q = \mathbb{Z}_q[x]/(x^n + 1)$:

$$f \cdot g = \mathbf{f}\mathrm{G} = \mathbf{g}\mathrm{F}$$

where $\mathrm{F}, \mathrm{G} \in \mathbb{Z}_q^{n \times n}$, whose columns are rotations of $\mathbf{f}, \mathbf{g}$, with possibly opposite sign:

$$\mathrm{F} = \begin{bmatrix} f_0 & -f_{n-1} & \dots & -f_1 \\ f_1 & f_0 & \dots & -f_2 \\ \dots & \dots & \dots & \dots \\ f_{n-1} & f_{n-2} & \dots & f_0 \end{bmatrix}$$

# BLISS Lattice-based Signature Scheme

- Public key $\mathbf{A} \in R_{2q}^{2\times 1}$, secret key $\mathbf{S} \in R_q^{1\times 2}$
- Secret vectors are sparse (entries typically in $\{1, -1, 0\}$).
- Attacker can validate correctness for candidate of key $\mathbf{S}$ with the public key, requiring only the first secret key $\mathbf{s}_1$.
- Both $-\mathbf{S}$ and $\mathbf{S}$ are valid as secret key.

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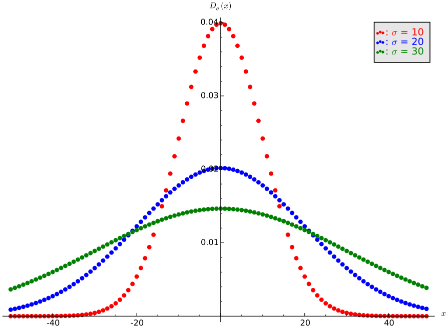
- $\mathbf{s}_1 \cdot \mathbf{c} = \mathbf{s}_1 \mathrm{C}$ over $\mathbb{Z}$ for matrix $\mathrm{C} \in \{-1,0,1\}^{n \times n}$.
- Equation hidden in signature over $\mathbb{Z}$:

$$\mathbf{z}_1 = \mathbf{y}_1 + (-1)^b \mathbf{s}_1 \mathrm{C}$$

where the unknowns for the attacker are $\mathbf{y}_1, b, \mathbf{s}_1$

# Discrete Gaussian Distribution




- Step 1 in signature algorithm: $\mathbf{y} \leftarrow D_{\mathbb{Z}^m,\sigma}$
- This is required to achieve (provable) security.
- Not straightforward to do in practice: high precision required.
- But how do we use additional knowledge of $\mathbf{y}$ to find $\mathbf{s}$?

# Attack Scenario's

# Attack Scenario 1

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Scenario 1:
We can determine $\mathbf{y}$ completely from a side-channel attack

- Only need one signature.
- Solve equation $(-1)^b(\mathbf{z} - \mathbf{y}) = \mathbf{s}\mathrm{C}$ for $\mathbf{s}$.
- But unrealistic...(?)

# Attack Scenario 2

- System of $n$ equations over $\mathbb{Z}$:

$$\underbrace{\begin{bmatrix} z_0 \\ z_1 \\ \dots \\ z_{n-1} \end{bmatrix}}_{\text{Signature 1}} = \underbrace{\begin{bmatrix} y_0 \\ y_1 \\ \dots \\ y_{n-1} \end{bmatrix}}_{\text{Noise}} + \underbrace{(-1)^b}_{\text{Sign}} \underbrace{\begin{bmatrix} - & \mathbf{c}_0 & - \\ - & \mathbf{c}_1 & - \\ - & \dots & - \\ - & \mathbf{c}_{n-1} & - \end{bmatrix}}_{\text{Challenge}} \cdot \underbrace{\begin{bmatrix} s_0 \\ s_1 \\ \dots \\ s_{n-1} \end{bmatrix}}_{\text{Secret}}$$

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Scenario 2:

There is a small set of values and an attacker can determine $y_i$ when it is in this set.

- Since this set is small, we need more than one signature.
- Zoom in on coordinate-wise equations:

$$z_i = y_i + (-1)^b \langle \mathbf{c}_i, \mathbf{s} \rangle$$

- If we know $y_i$, we save $\zeta_k = \mathbf{c}_i$ in a list.

# Attack Scenario 2

- If we know $y_i$, we save $\zeta_k = \mathbf{c}_i$ in a list.
- We can acquire enough of these vectors from multiple signatures and form:

$$\begin{bmatrix} (-1)^{b_0}(z_0 - y_0) \\ (-1)^{b_0}(z_0 - y_0) \\ \dots \\ (-1)^{b_{n-1}}(z_{n-1} - y_{n-1}) \end{bmatrix} = \begin{bmatrix} - & \zeta_0 & - \\ - & \zeta_1 & - \\ - & \dots & - \\ - & \zeta_{n-1} & - \end{bmatrix} \cdot \begin{bmatrix} s_0 \\ s_1 \\ \dots \\ s_{n-1} \end{bmatrix}$$

- Unfortunately: all bits $b_i$ are unknown.

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- Trick: if we know $y_i$, we can *be selective* that $z_i = y_i$, before saving $\zeta_k = \mathbf{c}_i$ in our list.
- We can eliminate $b$:

$$(-1)^b (z_i - y_i) = 0 = \langle \zeta_k, \mathbf{s} \rangle$$

# Attack Scenario 2

Scenario 2:
There is a small set of values and an attacker can determine $y_i$ when it is in this set.

- If we know $y_i$ and $z_i = y_i$: we save $\zeta_k = \mathbf{c}_i$ in a list $\mathrm{M}$.
- Acquire enough of these vectors from multiple signatures and we have equation:

$$\mathbf{s}\mathrm{L} = \mathbf{o}$$

- With very high probability: secret vector $\mathbf{s}$ is the only vector in the integer (left) kernel of $\mathrm{L}$.
- Calculating the integer kernel is easy.

# Attack Scenario 3

- Signature equation over $\mathbb{Z}$: $\mathbf{z} = \mathbf{y} + (-1)^b \mathrm{C}\mathbf{s}$.
- Let us go one step further:

Scenario 3:
There is a small set of tuples $\{\gamma, \gamma + 1\}$ and an attacker can determine the tuple for $y_i$ when it is in this set.
**With high probability,** $y_i = \gamma$

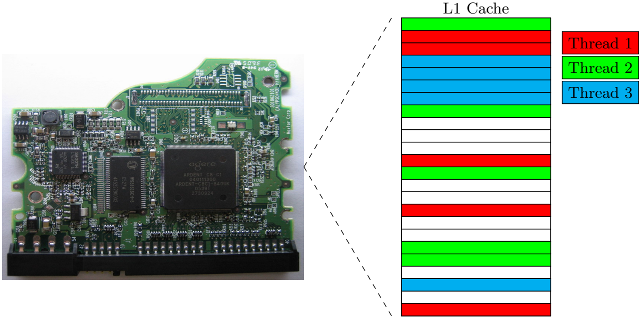
# Attack Scenario 3

- Apply same method as previous:
- If we know $\color{red}{y_i}$ $\in \{\gamma, \gamma + 1\}$ and $z_i = \gamma$: we save $\zeta_k = \mathbf{c}_i$ in a list $\mathrm{M}$.
- Now $\mathbf{s}\mathrm{L}$ is not an all-zero vector, but it is small.
- Use LLL-algorithm to compute small vectors, search for **s** in the unitary transformation matrix.
- Verify correctness with public key.

# Cache Timing Attacks

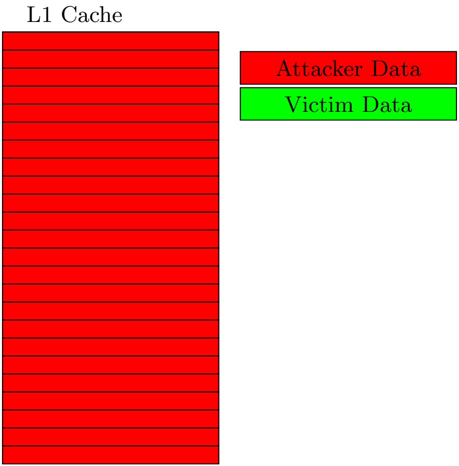
# Cache (Timing) Attacks

- Cache-memory: small, fast memory shared among all threads.
- Bridge the gap between processor speed and main memory RAM speed .
- Data is stored in cache-lines, typically 64 Bytes.



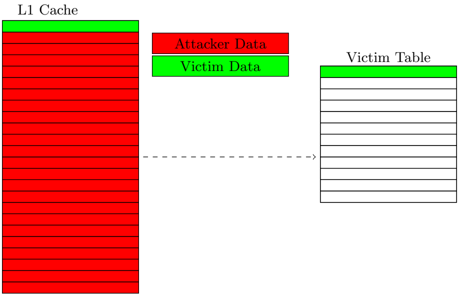
# Cache (Timing) Attacks

- Attacker fills cache with his data.



# Cache (Timing) Attacks

- Attacker notices that victim uses some part of cache.
- Learns cache-line of data used by victim.

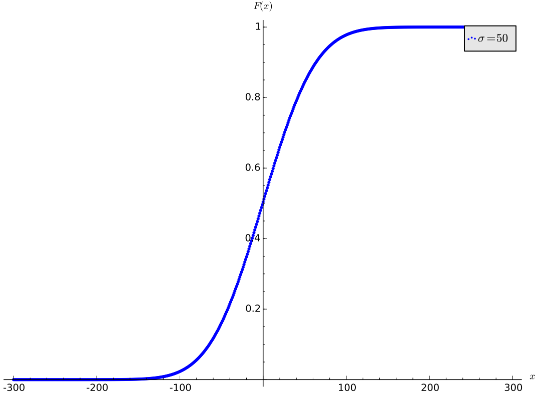


# Cache-attacks on BLISS

- $\mathbf{y} \leftarrow D_{\mathbb{Z}^m,\sigma}$
- Three attack scenario's using additional knowledge of **y**.
- Implemented cache-attacks on two discrete Gaussian samplers: CDT sampling and Rejection-based sampling, which both use table look-ups.

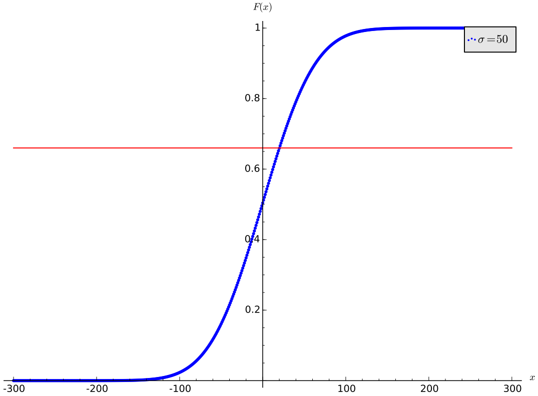
# Cache-Attacking BLISS with CDT Sampling

# CDT Sampling with Guide Table



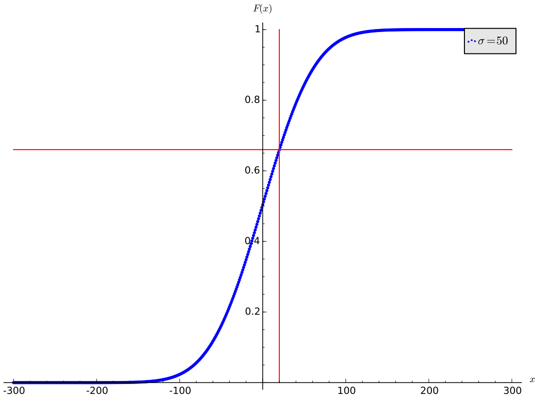

1. Save values of the discrete Gaussian CDF in table $T$.

# CDT Sampling with Guide Table




2. Generate a random value $r \in [0, 1)$

# CDT Sampling with Guide Table




3. Perform a binary search to find sample $x$ with $T(x-1) \leq r < T(x)$.

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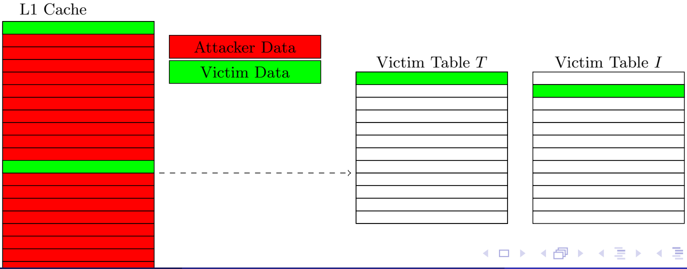

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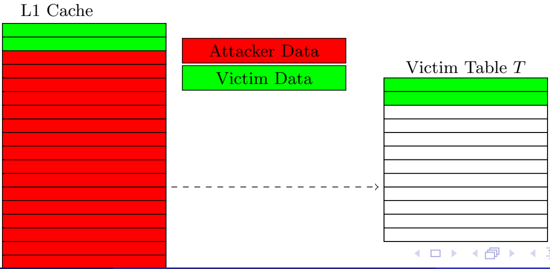
- Two types of cache weaknesses:
  - Intersection (use knowledge of accesses in $I$ and $T$)
  - Last-jump (track the binary search using knowledge of multiple accesses in $T$)



# Cache-attack BLISS with CDT Sampling

- Hidden equation in signature: $\mathbf{z} = \mathbf{y} + (-1)^b \mathbf{s}\mathrm{C}$.
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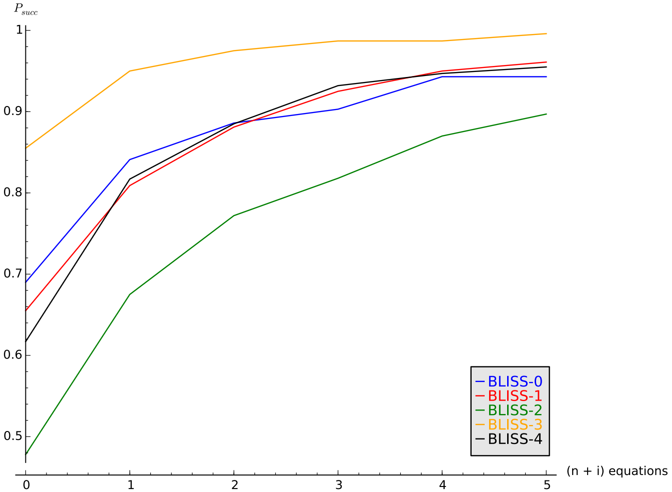
- Attacker can now apply strategy for scenario 3.
- Collect more signatures and randimize if necessary: greatly increases success probability for the attack to find the secret key.

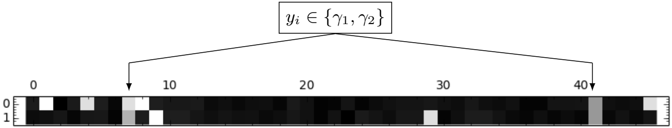
# Experiments

- Results (modelled) cache-attack with perfect side-channel.
- BLISS with CDT sampling:



# Experiments

- Proof-of-concept attack using FLUSH+RELOAD technique.
- Visualization of last-jump weakness:




- Experiments with BLISS-I succeeded 90% of the time.

# Experiments

- Similar method and results achieved for rejection-based sampling method.
- Details in paper at *https://eprint.iacr.org/2016/300*.

# Conclusions

- The discrete Gaussian sampler leaks enough information via cache-memory to find a BLISS secret key.
- But these attacks require significant power over device, does not effect theoretical security of BLISS.
- However, more research is needed to get security for implementations.

Thank you for your attention!
Are there any questions?